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SUBSETS

Suppose A and B are two sets. A is a **subset** of B if every element of A is also an element of B. We write $A \subseteq B$.

For example:

- If $E = \{\text{even numbers}\}$ then $E \subseteq \mathbb{Z}$.
- The empty set $\varnothing$ is a subset of every set.

Example 3 **Self Tutor**

Suppose $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 3, 5\}$, and $C = \{3, 5, 8\}$.

Decide whether B or C are subsets of A.

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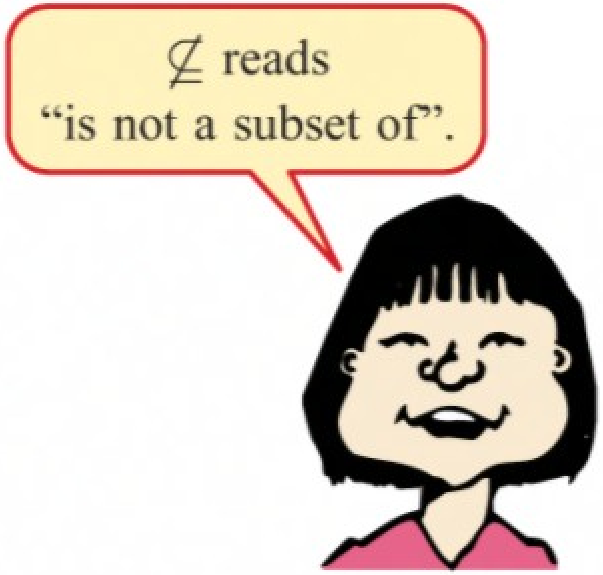
Decide whether B or C are subsets of A.

Every element of B is also an element of A, so $B \subseteq A$.

The element 8 of C is not an element of A, so $C \nsubseteq A$.


$\nsubseteq$ reads "is not a subset of".

THE COMPLEMENT OF A SET

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The complement of A, denoted A', is the set of all elements of U which are *not* in A.

$$A' = \{x \mid x \notin A,\ \ x \in U\}$$

Example 4 **Self Tutor**

Suppose $U = \{x \mid x \leqslant 12,\ x \in \mathbb{Z}^+\}$. Find the complement of:

a $A = \{\text{even numbers in } U\}$

b $B = \{\text{prime numbers in } U\}$

Example 4 **Self Tutor**

Suppose $U = \{x \mid x \leqslant 12,\ x \in \mathbb{Z}^+\}$. Find the complement of:

a $A = \{\text{even numbers in } U\}$

b $B = \{\text{prime numbers in } U\}$

a $A' = \{\text{odd numbers in } U\}$
$= \{1, 3, 5, 7, 9, 11\}$

b $B' = \{1, 4, 6, 8, 9, 10, 12\}$

EXERCISE 2C

1 For each of the following sets A and B, decide whether $A \subseteq B$:

a $A = \{2, 5, 6\}, \quad B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

b $A = \{4, 8, 11, 12\}, \quad B = \{2, 4, 6, 8, 10, 12, 14, 16\}$

c $A = \varnothing, \quad B = \{1, 4, 7, 10\}$

d $A = \{5, 10, 15, 20, 25, 30\}, \quad B = \{10, 15, 20\}$

e $A = \{6, 7, 8\}, \quad B = \mathbb{N}$

2 Are the following statements true or false?

a $\mathbb{Z} \subseteq \mathbb{R}$

b $\mathbb{Z}^+ \subseteq \mathbb{Z}$

c $\{\frac{1}{3}, \sqrt{2}, 5\} \subseteq \mathbb{Q}$

d $\mathbb{N} \subseteq \mathbb{Q}$

e $\mathbb{R} \subseteq \mathbb{Z}$

f $\mathbb{Z}^+ \subseteq \mathbb{N}$

3 Suppose $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{3, 6, 9\}$, and $C = \{4, 7\}$. Decide whether B or C are subsets of A.

4 Suppose $P = \{\text{prime numbers less than } 10\}$, $Q = \{\text{multiples of 3 less than } 20\}$, $R = \{3, 5, 7\}$, and $S = \{\text{multiples of 6 less than } 20\}$. Decide whether the following statements are true or false:

a $P \subseteq Q$

b $R \subseteq P$

c $R \subseteq S$

d $S \subseteq Q$

5 Suppose $U = \{x \mid x \leqslant 9,\ x \in \mathbb{Z}^+\}$. Find the complement of:

a $A = \{2, 5, 6\}$

b $B = \{\text{prime numbers in } U\}$

c $C = \{\text{odd numbers in } U\}$

d $D = \{\text{multiples of 4 in } U\}$

e $E = \varnothing$

f $F = \{x \mid x < 3,\ x \in \mathbb{Z}^+\}$.

6 Suppose $U = \{\text{letters of the English alphabet}\}$. Find the complement of:

a $P = \{\text{C, F, J, M, P, U, Y, Z}\}$

b $Q = \{\text{consonants}\}$

c $R = \{\text{letters in the word HOSPITAL}\}$

d $S = \{\text{letters after J in the alphabet}\}$.

7 Suppose $U = \{x \mid x \leqslant 15,\ x \in \mathbb{Z}^+\}$, $A = \{x \mid 5 \leqslant x < 13,\ x \in \mathbb{Z}^+\}$, and $B = \{x \mid 6 < x < 10,\ x \in \mathbb{Z}^+\}$.

a Write down A' and B'.

b True or false?

i $A \subseteq B$

ii $B \subseteq A$

iii $A' \subseteq B'$

iv $B' \subseteq A'$

1 **a** $A \subseteq B$ **b** $A \nsubseteq B$ **c** $A \subseteq B$ **d** $A \nsubseteq B$
e $A \subseteq B$

2 **a** true **b** true **c** false **d** true **e** false **f** true

3 $B \nsubseteq A,\ C \subseteq A$

4 **a** false **b** true **c** false **d** true

5 **a** $A' = \{1, 3, 4, 7, 8, 9\}$ **b** $B' = \{1, 4, 6, 8, 9\}$
c $C' = \{2, 4, 6, 8\}$ **d** $D' = \{1, 2, 3, 5, 6, 7, 9\}$
e $E' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (or $E' = U$)
f $F' = \{3, 4, 5, 6, 7, 8, 9\}$

6 **a** $P' = \{$A, B, D, E, G, H, I, K, L, N, O, Q, R, S, T, V, W, X$\}$
b $Q' = \{$vowels$\}$
c $R' = \{$B, C, D, E, F, G, J, K, M, N, Q, R, U, V, W, X, Y, Z$\}$
d $S' = \{$A, B, C, D, E, F, G, H, I, J$\}$

7 **a** $A' = \{1, 2, 3, 4, 13, 14, 15\}$
$B' = \{1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15\}$

b **i** false **ii** true **iii** true **iv** false